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H_∞ CONTROL OF A GRAVITATIONAL WAVE DETECTOR USING NORMALIZED COPRIME FACTOR PLANT DESCRIPTIONS

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Abstract: This paper deals with large-scale gravitational-wave detector control. These detectors use multiple optical cavities and require a coherent control of several suspended mirror positions. In fact the mirrors forming the detector are oscillating around their equilibrium positions under the influence of the earth seismic noise. Hence a control system is needed in order to control the mirror positions at a certain precision level specified by the detection sensitivity.
We present in this paper a H_∞ control approach of the detector, based on the H_∞ loop shaping design method of Glover and Mc-Farlane(D.McFarlane and K.Glover, 1992). The design method is first discussed and then the simulation results, obtained with the suspension simulation program, are commented. *Copyright © 1998 IFAC*

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1. INTRODUCTION

Several gravitational wave detectors based on Michelson interferometers using long Fabry-Perot cavities are under construction (VIRGO collaboration, 1997; LIGO Collaboration, 1992). The mirrors making up the interferometer will be suspended masses (figure 1) oscillating around their equilibrium points at very low frequency. The amplitude of the oscillations exceeds by as much as six orders of magnitude the required stability on the distances between the mirrors for the interferometer to be operational. A complex control system is therefore needed to reduce the amplitude of the mirror oscillations in order to meet the various conditions of resonance and interference defining the working point. This control system will use signals collected by several photo-diodes located at various places in the interferometer which provide information about the mirrors relative positions (see (C.Mehmel *et al.*, 1998) for the modelization procedure). In order to achieve the detection sensitivity four characteristic lengths representing relative distances between the interferometer mirrors should be controlled. In fact, this becomes to control four mirror positions.
We focus in this paper on the control of a single suspended mirror.

The controller has been designed according to the Glover-McFarlane H_∞ loop shaping method. The main motivation of using this approach is that it generally leads to controllers that do not systematically cancel the dynamic of the plant. This point is very important in our case since the suspension model has several lightly damped modes. The method is also desirable for multivariable systems since it allows to trade-off between stability margins at plant input and output.

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2. $H\infty$ LOOP-SHAPING DESIGN METHOD

The $H\infty$ loop-shaping method based on coprime factor representation introduced in (D.McFarlane and K.Glover, 1992), has several attractive properties. Indeed this method unlike the mixed sensitivity approach (H.Kwakernaak, 1993) or the LQG/LTR (G.Stein and M.Athans, 1987; C.Mehmel *et al.*, 1997) approach avoids exact pole-zero cancellation between the plant and the controller.

Another attractive property of this method is that it does not require the usual γ-iterations of the H_∞ problem as stated in (J.C.Doyle *et al.*, 1989). Indeed in this case the optimal stability margin is known a priori. In the following we will summarize the main results leading to the H_∞ controller construction. The nominal plant is represented by a normalized left coprime factorization. Hence the plant model can be written as:

$$\mathbf{G} = \mathbf{M}^{-1} \cdot \mathbf{N} \tag{1}$$

where

$$N \cdot N^* + M \cdot M^* = I \tag{2}$$

In the above formula, M and N are stable, rational transfer matrices. A method to obtain a minimal order factorization can be found in (D.McFarlane and K.Glover, 1990). If we define now the class of all the model perturbations as:

$$\mathcal{D}_{c\epsilon} \triangleq \{\Delta = [\Delta_N, \Delta_M]; \Delta \in RH_\infty, \|\Delta\|_\infty < \epsilon\} \tag{3}$$

Where RH_∞ denotes the set of all stable rational transfer matrices, having no poles on the imaginary axis. The perturbed model is given by:

$$\mathbf{G} = (\mathbf{M} + \Delta_\mathbf{M})^{-1} \cdot (\mathbf{N} + \Delta_\mathbf{N}) \tag{4}$$

The largest positive number ϵ_{max} such that the closed-loop remains stable for all the perturbation of the class $\mathcal{D}_{c\epsilon_{max}}$ is given by:

$$\epsilon_{max} = (inf_K \|[K, I]^T \cdot (I - G \cdot K)^{-1} M^{-1}\|_\infty)^{-1}$$

This formula can be derived from the small gain theorem. The main result of the theory is that the largest bound on the allowed perturbations is known a priori and given (see (D.McFarlane and K.Glover, 1990)) by:

$$\epsilon_{max} = (1 - \|[NM]^T\|_H^2)^{\frac{1}{2}} \tag{5}$$

Where $\|\cdot\|_H$ denotes the Hankel norm and X^T the transpose of the matrix X. The controller for a stability margin arbitrarily close to ϵ_{max} is constructed by solving two LQG-type Riccati equations (see (D.McFarlane and K.Glover, 1990)).

Since the design of the compensator does not use any weighting functions the loop-shaping design is made as follows:

- A pre- and (or) a post-compensation is performed in order to achieve the performance requirements
- A stabilizing compensator for the shaped plant is computed according to the procedure above
- The value of ϵ_{max} is an indicator on how the choice of the first compensator agrees with robustness objectives. Hence a small value of ϵ_{max} makes the redesign of the first compensation mandatory. Usually the design is acceptable when $\epsilon_{max} > 0.25$.

3. PLANT MODEL AND DESIGN OBJECTIVES

In order to achieve the detection sensitivity each of the detector mirrors is suspended as shown in figure (1). The suspension is constituted by seven mechanical filters and a last stage called marionette. A system of coils and magnets fixed on the marionette allows to control the mirror position. Another action mode is given by a reference mass supported as the mirror by the marionette. A coil is placed on the reference mass and a permanent magnet is fixed directly on the mirror. A current applied to the coil induces a repulsive force between the mirror and the reference mass and then controls the mirror position.

The advantage of this action mode is that the model relating the applied current and the mirror displacement is quite simple (second order transfer function). This comes from the fact that the dynamic of the upper stages is not excited when we act through the reference mass. However the design of the reference mass makes a limit to the maximal displacement that can be achieved by this control mode. Hence the reference mass can not be used at low frequency (up to 1Hz) where the RMS open loop motion of the mirror is quite important. The figures (2) and (3) show the bode plots of the transfer functions relating both the marionette and the reference mass control signal to the mirror position.
Another requirement is that the unity gain frequency on the marionette should be kept as small as possible. This requirement comes from the fact that the marionette transfer function is not well known. The table (1) gives the design objectives of the control system. These requirements are given for a mirror vibration amplitude of $1\mu m$.

The open loop maximal displacement equals $1\mu m$.

Table 1. Control Requirements.

Closed loop maximal displacement	$10^{-6}\mu m$
Maximal force on the reference mass	$50\mu N$
Maximal force on the marionette	$20mN$
Bandwidth	< 100Hz

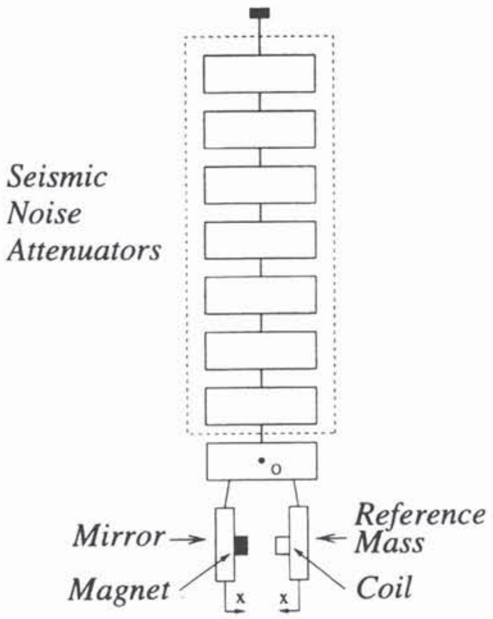

Fig. 1. *Suspension System*

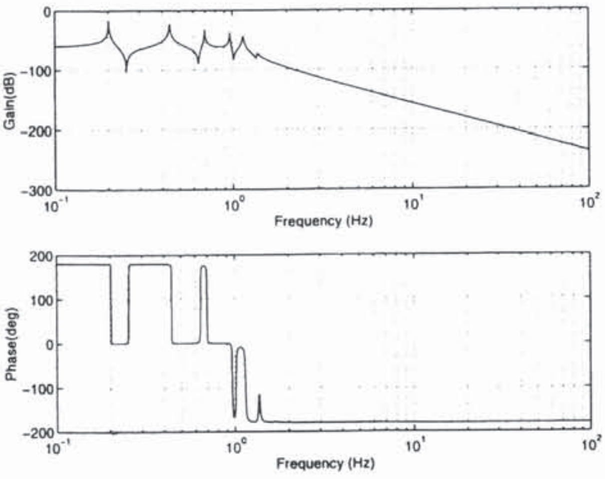

Fig. 2. *Marionette transfer function bode plot*

4. CONTROLLER DESIGN AND ROBUSTNESS ANALYSIS

The control strategy we present in this paper involves the use of the marionette action at low frequencies (up to Hz) and the reference mass action beyond this frequency range. This strategy allows to achieve the requirement on the reference mass force and the marionette unity gain frequency. The controller design procedure is as follows:

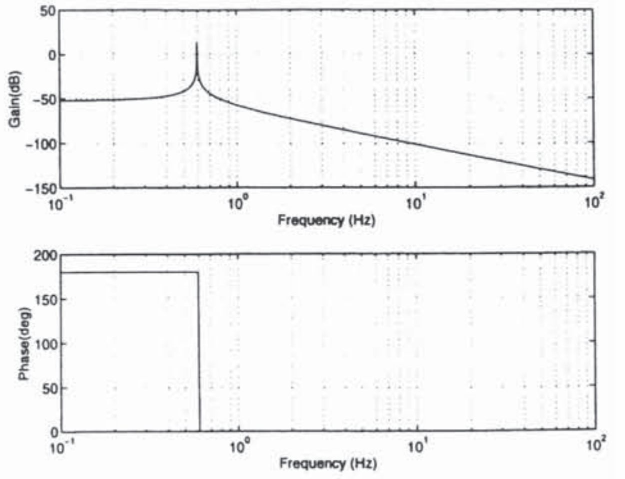

Fig. 3. *Reference mass transfer function bode plot*

- An integral action is used on the marionette in order to achieve the required seismic noise attenuation. The integral action is compensated at 0,1Hz in order to avoid the crossings of the marionette transfer function gain with the 0dB axis. Such crossings can lead the closed loop system to instability. The first anti-resonant mode in the marionette transfer function is slightly compensated in order to increase the open loop gain at this mode frequency. An another side a lead lag controller is used on the reference mass. This controller lowers the gain at low frequencies in such a way that the force applied through this actuator does not exceed the imposed limit.

$$W(s) = \begin{bmatrix} -5.e5\dfrac{s}{1+.159s} \\ -1.e8\dfrac{(1+1.5923s)}{s}\dfrac{1+.881s+.396s^2}{1+.00409s+.396s^2} \end{bmatrix}$$

- The suspension has a 20 order model. In order to reduce the order of the stabilizing compensator and to avoid numerical problems, a 8 order reduced model has been used.
- The use of the pre-compensator given above leads to a value of $\epsilon_{max} = 0.275$. However this value has been relaxed by a factor of 10% when computing the compensator in order to avoid numerical problems

The figures (4) and (5) show the bode plot of the whole compensator transfer functions.

The open-loop transfer function bode plot when the loop is broken at the plant output is given in figure (6). One can see that the bandwidth equals 50Hz. The output sensitivity versus the seismic noise is given in figure (7), the figure shows that the expected attenuation at 0.1Hz equals to 120 dB.

The figures (8) and (9) show the bode plot of the transfer functions obtained when breaking the loop respectively at the reference mass and the marionette. The marionette unity gain frequency is approximatively 12Hz.

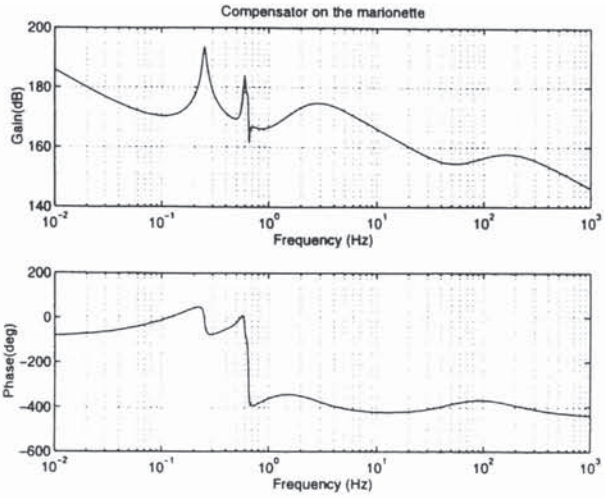

Fig. 4. *Compensator on the marionette bode plot*

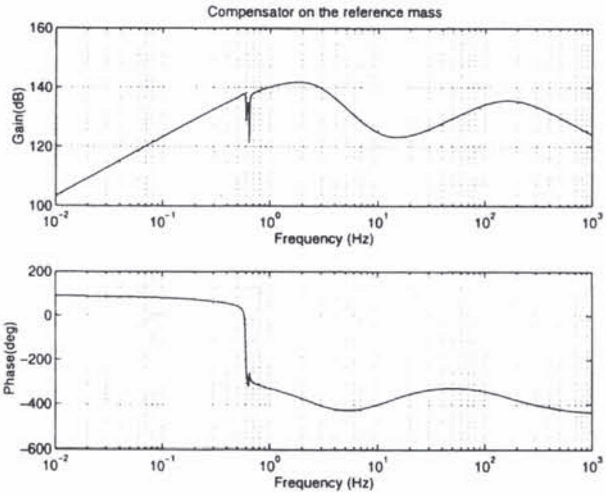

Fig. 5. *Compensator on the reference mass bode plot*

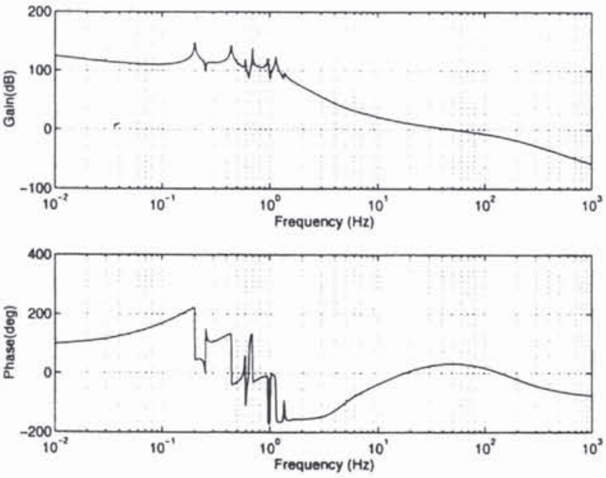

Fig. 6. *Output open-loop transfer function bode plot*

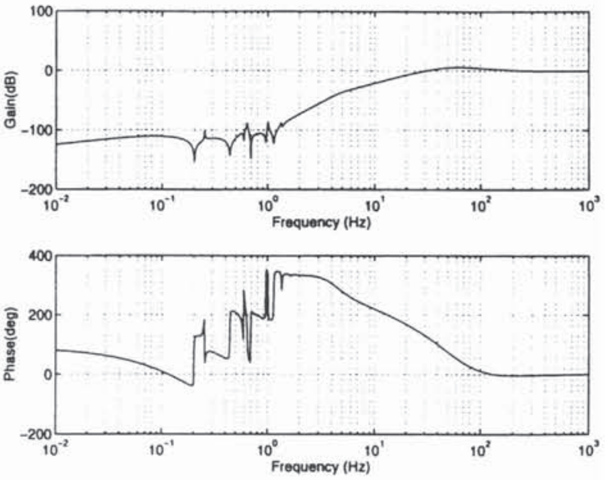

Fig. 7. *Sensitivity to output noise transfer function bode plot*

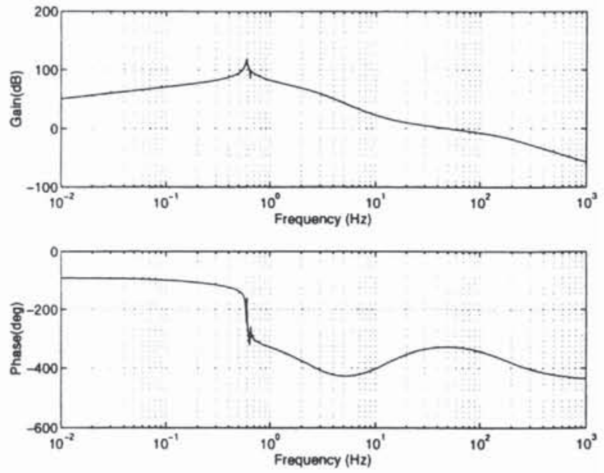

Fig. 8. *Input (1) open-loop transfer function bode plot*

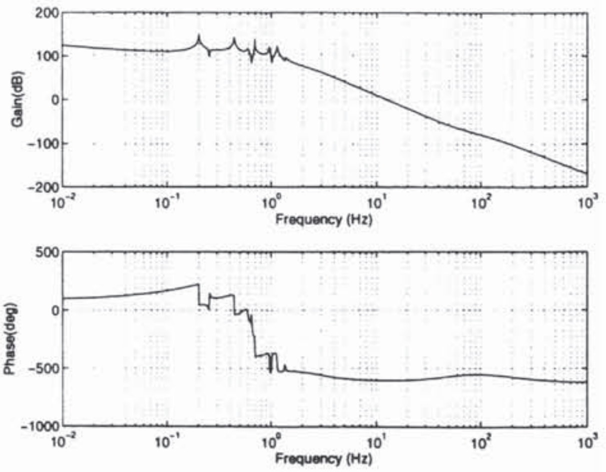

Fig. 9. *Input (2) open-loop transfer function bode plot*

A stability robustness analysis of the controlled system against uncertainties in the suspension model was performed. In order to compute the multivariable stability margins at the plant output, the μ analysis framework (J.C.Doyle, 1982) was used. The uncertainties have been represented as two block direct multiplicative input uncertain-

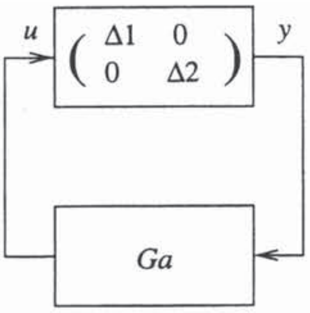

Fig. 10. *Standard robust stability problem*

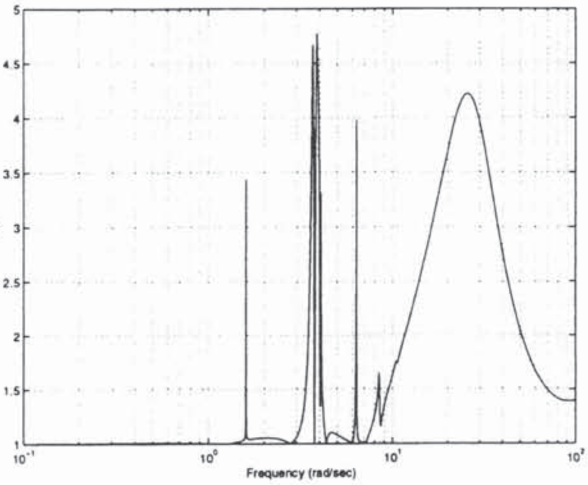

Fig. 11. *μ for input two block direct multiplicative uncertainties*

ties. This problem can be brought to the standard problem of figure (10).

The figure (11) shows the μ plot versus frequency.

The suspension model (B.Caron and M.Giordano, 1994) is derived using three sets of parameters: the suspension wires quality factors, the filter masses and the mechanical stage lengths. The most crucial parameters in the model derivation are the quality factors of the suspensions since the suspension transfer function are very sensitive to changes in these parameters.
The robustness analysis against uncertainties in the quality factors values was as follows:

- A sequence of 10000 parameter samples was generated by randomly disturbing the values of the quality factors within a fixed range and by keeping other parameters fixed.
- The stability of the closed system is checked, and the variation range is extended depending on whether the closed loop is stable for all the samples or not.
- The largest variation range is obtained when any extension of a stabilizing set of parameters leads to instability.

Notice here that the previous analysis method does not allow to obtain exact bounds on the model parameters. Also the analysis needs some a priori knowledge on the parameters uncertainty bounds, and it can be cumbersome when the number of parameters is high. The tables (2) and (3) show the stability margins obtained at the plant input with the H_∞ controller and a classical controller designed to achieve objectives at the plant output. One can see that the H_∞ improves the stability margins. The table (4) gives the quality factors allowed variation domain.

Table 2. H_∞ controller.

Gain margin	$[-2.03dB, 2.04dB]$
Phase margin	$[-12.014deg, 12.014deg]$

Table 3. Classical controller.

Gain margin	[-1.01dB,0,906dB]
Phase margin	[-6.14deg,12.014deg]

Table 4. Parameter variation ranges.

Parameter	Nominal value	Range
Quality factor 1	$f_1 = 100$	$[50, 140]$
Quality factor 2	$f_2 = 1e6$	$[1e4, 1e7]$

5. SIMULATION RESULTS

The controller has been tested with the suspension simulation program. The figures (12) and (13) show the open-loop variation and closed variation of the mirror position. The control signals on the reference mass and the marionette are shown respectively on the figures (14) and (15).

The simulation results are summarized in table (5):

Table 5. Simulation results

	max	rms
Reference mass force	$1.8 \cdot 10^{-6}$ N	$5.521 \cdot 10^{-7}$ N
Marionette force	$9.162 \cdot 10^{-4}$ N	$2.834 \cdot 10^{-4}$ N
Closed loop displacement	$5.656 \cdot 10^{-13}$ m	$1.446 \cdot 10^{-13}$ m

6. CONCLUSION

In this paper we have presented an H_∞ approach for a gravitational wave detector control. The controller designed using the normalized plant factorization approach allows to meet the design requirements. The main advantage of the method is that the controller does not systematically cancel the dynamic of the plant.

The robustness analysis has shown that the closed loop system remains stable for large variations of the suspension wires quality factors.

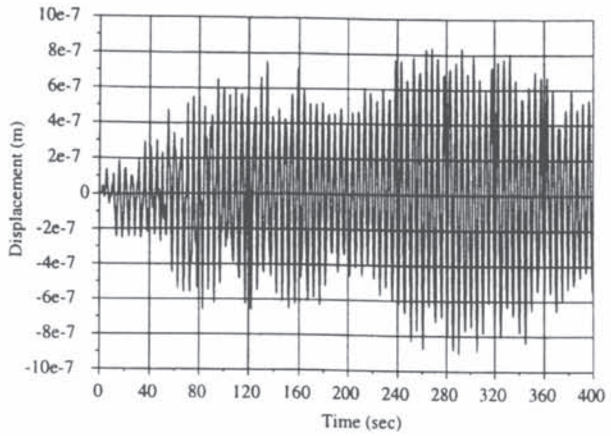

Fig. 12. *Mirror open loop variation.*

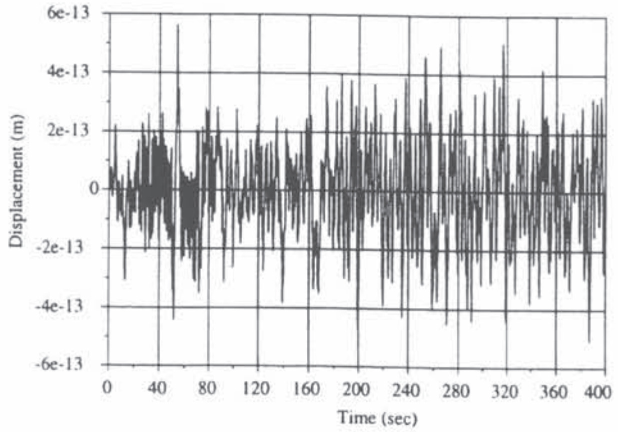

Fig. 13. *Mirror closed loop variation.*

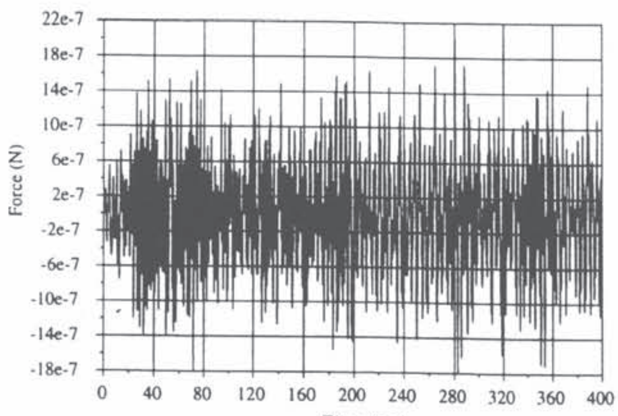

Fig. 14. *Control signal on the reference mass.*

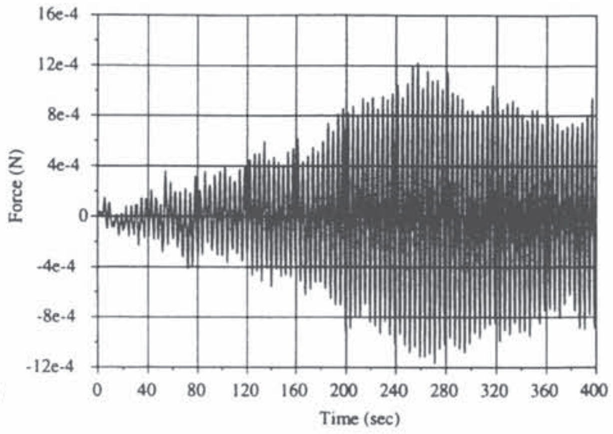

Fig. 15. *Control signal on the marionette.*

Other design methods allowing improvement of the stability margins and compensator order reduction should be investigated.

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